

TRANSFERÊNCIA DE CALOR

Terças e Quintas de 8:00 às 10:00 hs

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Lab CFD

Aula 11

❖ Condução Transiente - Efeitos Multidimensionais

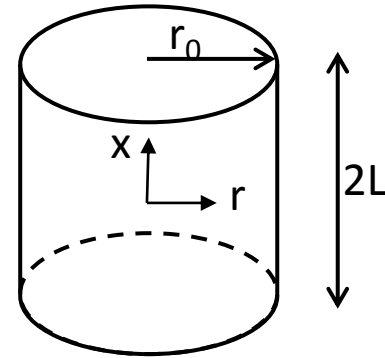
- ❖ Método da Separação das Variáveis
- ❖ Cilindro Curto
- ❖ Cilindro Semi-Infinito
- ❖ Placa Semi-Infinita
- ❖ Barra Retangular Infinita
- ❖ Barra Retangular Semi-Infinita
- ❖ Paralelepípedo
- ❖ Exemplo

Exercícios do Cap. 5: 7, 10, 11, 15, 33, 44, 50, 55, 60, 66, 71, 79, 87, 88, 89, 90 e 91

Efeitos Multidimensionais

Suponha a imersão de um cilindro curto, inicialmente a uma temperatura T_i , em um fluido de temperatura T_∞ .

$$\frac{1}{\alpha} \frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial x^2} \Rightarrow \frac{1}{\alpha} \frac{\partial \theta^*}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \theta^*}{\partial r} \right) + \frac{\partial^2 \theta^*}{\partial x^2}$$



Condição inicial: $T(r, x, 0) = T_i \Rightarrow \theta^*(r, x, 0) = 1$

Condição de Simetria: $\left. \frac{\partial T}{\partial x} \right|_{x=0} = 0 \Rightarrow \left. \frac{\partial \theta^*}{\partial x} \right|_{x=0} = 0$

$$\left. \frac{\partial T}{\partial r} \right|_{r=0} = 0 \Rightarrow \left. \frac{\partial \theta^*}{\partial r} \right|_{r=0} = 0$$

Condição de Convecção: $-k \left. \frac{\partial T}{\partial x} \right|_{x=L} = h[T(r, L, t) - T_\infty] \Rightarrow -k \left. \frac{\partial \theta^*}{\partial x} \right|_{x=L} = h\theta^*(r, L, t)$

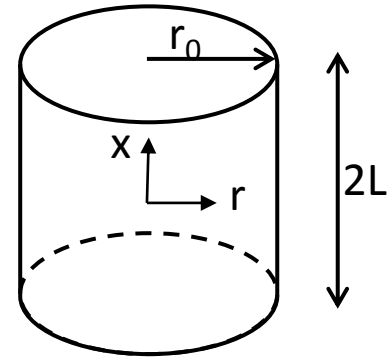
$$-k \left. \frac{\partial T}{\partial r} \right|_{r=r_0} = h[T(r_0, x, t) - T_\infty] \Rightarrow -k \left. \frac{\partial \theta^*}{\partial r} \right|_{r=r_0} = h\theta^*(r_0, x, t)$$

Efeitos Multidimensionais

Método da Separação das Variáveis: $\theta^*(r, x, t) = C(r, t)P(x, t)$

$$C(r, t) = \frac{T(r, t) - T_\infty}{T_i - T_\infty} \Big|_{\text{cilindro infinito}} \Rightarrow \theta^* \Big|_{\text{cilindro infinito}} \Rightarrow \theta_c^*$$

$$\theta_c^* = C_{1c} \exp(-\xi_{1c}^2 Fo_c) J_0(\xi_{1c} r^*) \quad Bi_c = \frac{hr_o}{k} \quad Fo_c = \frac{\alpha t}{r_o^2}$$



$$P(r, t) = \frac{T(x, t) - T_\infty}{T_i - T_\infty} \Big|_{\text{placa plana}} \Rightarrow \theta^* \Big|_{\text{placa plana}} \Rightarrow \theta_p^*$$

$$\theta_p^* = C_{1p} \exp(-\xi_{1p}^2 Fo_p) \cos(\xi_{1p} x^*) \quad Bi_p = \frac{hL}{k} \quad Fo_p = \frac{\alpha t}{L^2}$$

Efeitos Multidimensionais

Conferência de validação do método: $\theta^*(r, x, t) = C(r, t)P(x, t)$

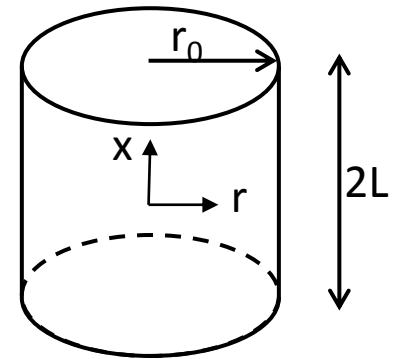
$$\frac{1}{\alpha} \frac{\partial \theta^*}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \theta^*}{\partial r} \right) + \frac{\partial^2 \theta^*}{\partial x^2}$$

$$\frac{1}{\alpha} \left(P \frac{\partial C}{\partial t} + C \frac{\partial P}{\partial t} \right) = P \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial C}{\partial r} \right) + C \frac{\partial^2 P}{\partial x^2}$$

$$P \left[\frac{1}{\alpha} \frac{\partial C}{\partial t} - \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial C}{\partial r} \right) \right] + C \left(\frac{1}{\alpha} \frac{\partial P}{\partial t} - \frac{\partial^2 P}{\partial x^2} \right) = 0$$

Cilindro Infinito: $\frac{1}{\alpha} \frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right)$

Placa Plana: $\frac{1}{\alpha} \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}$



Efeitos Multidimensionais

Sendo assim, definindo:

$$C(r,t) = \frac{T(r,t) - T_{\infty}}{T_i - T_{\infty}} \Big|_{\text{cilindro infinito}}$$

$r=0$ no centro do cilindro
 $0 \leq r \leq r_o$

$$P(x,t) = \frac{T(x,t) - T_{\infty}}{T_i - T_{\infty}} \Big|_{\text{placa plana}}$$

$x=0$ na meia espessura da placa
 $-L \leq x \leq L$

$$S(x,t) = \frac{T(x,t) - T_{\infty}}{T_i - T_{\infty}} \Big|_{\text{sólido semi-infinito Caso 3}}$$

$x=0$ na superfície
 $x \geq 0$

Podemos compor a solução para problemas bi e tridimensionais, que partem de uma temperatura inicial uniforme T_i e trocam calor por convecção na superfície com um fluido a T_{∞} , através do produto de soluções unidimensionais para essas mesmas condições.

Efeitos Multidimensionais

Observação: $S(x,t) = \frac{T(x,t) - T_\infty}{T_i - T_\infty}$ | sólido semi-infinito Caso 3

Sólido Semi-Infinito Caso 3:

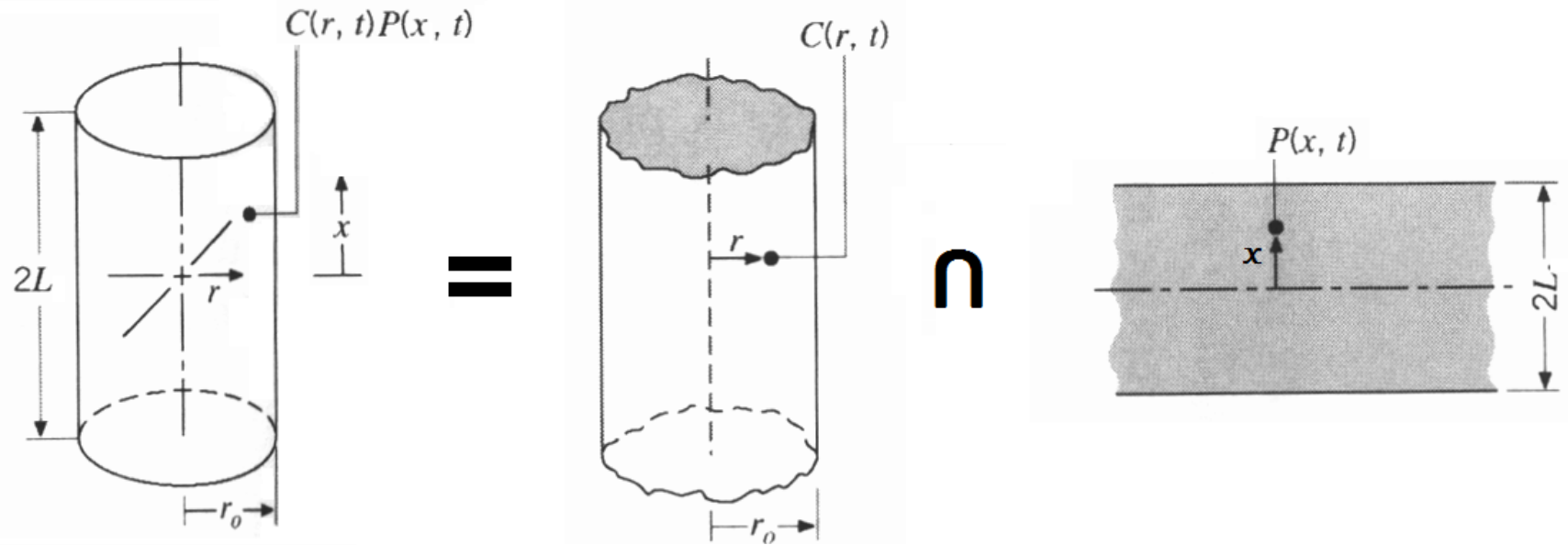
$$\frac{T(x,t) - T_i}{T_\infty - T_i} = \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \left[\exp\left(\frac{hx}{k} + \frac{h^2\alpha t}{k^2}\right)\right] \left[\operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right)\right]$$

$$\frac{T(x,t) - T_i}{T_\infty - T_i} = 1 - \frac{T(x,t) - T_\infty}{T_i - T_\infty} = 1 - S(x,t)$$

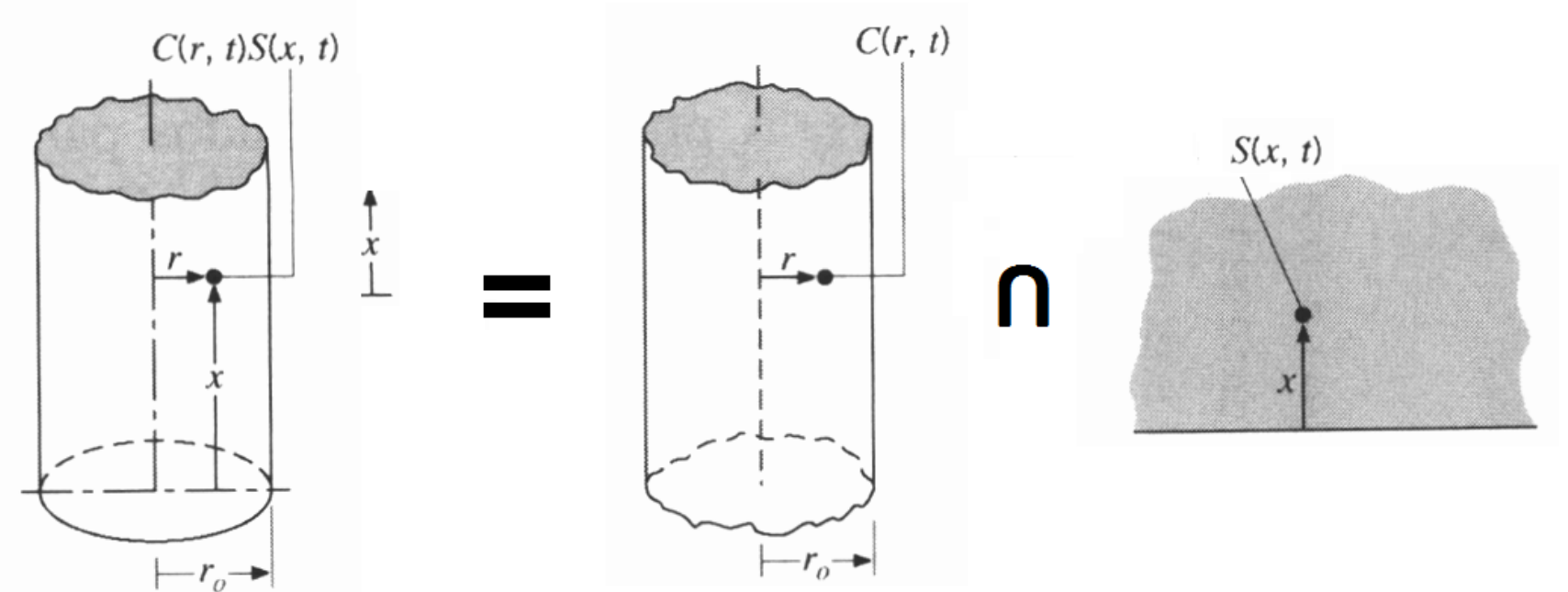
$$S(x,t) = 1 - \frac{T(x,t) - T_i}{T_\infty - T_i}$$

Efeitos Multidimensionais

Cilindro Curto

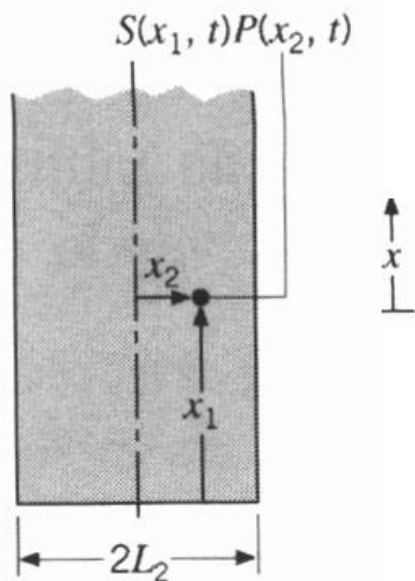


Cilindro Semi-Infinito

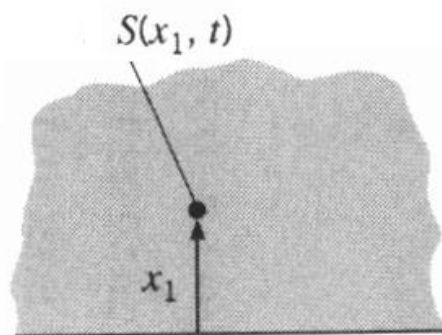


Efeitos Multidimensionais

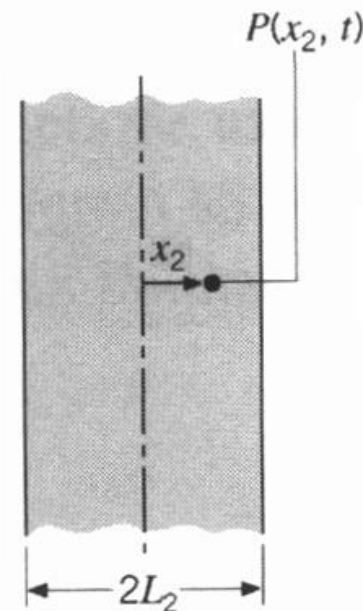
Placa Semi-Infinita



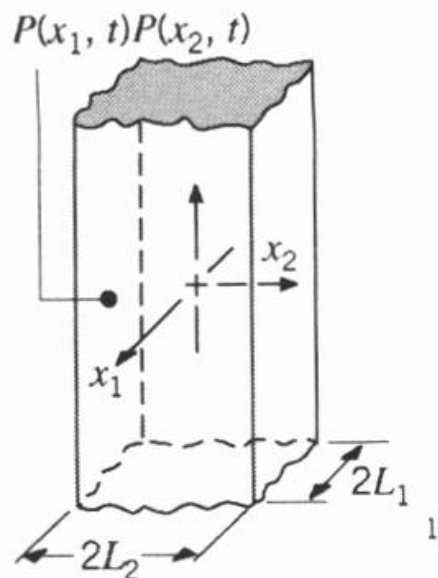
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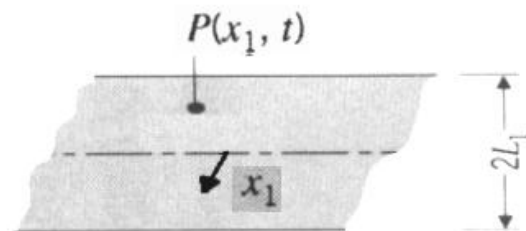
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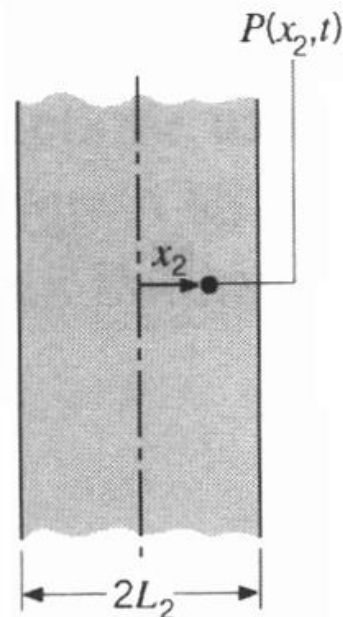
Barra Retangular Infinita



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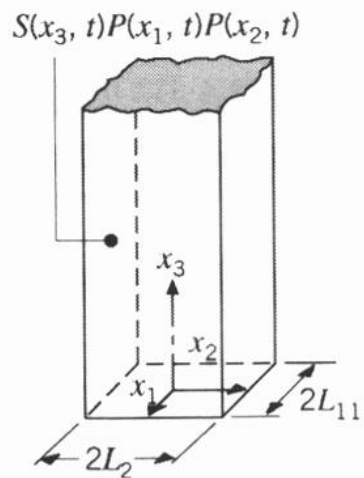


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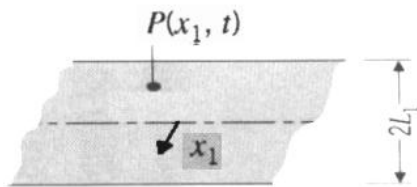


Efeitos Multidimensionais

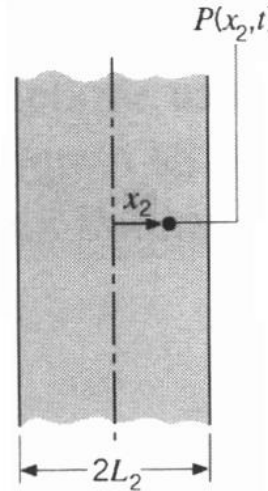
Barra Retangular Semi-Infinita



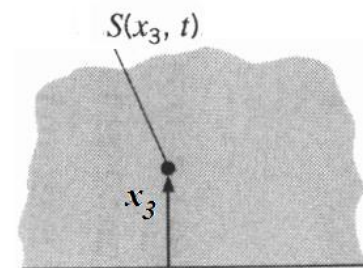
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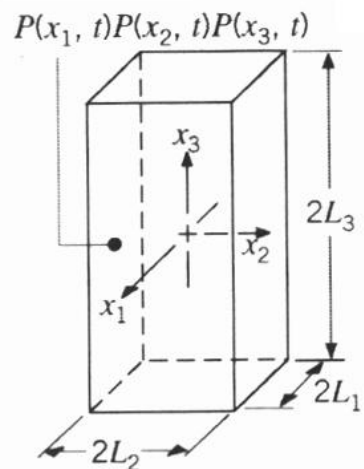
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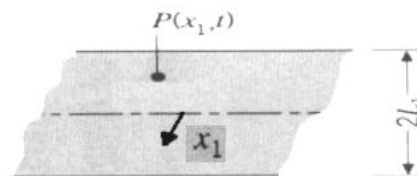
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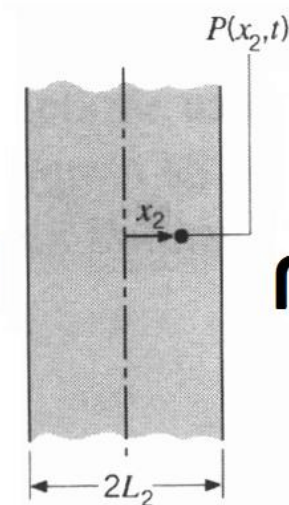
Paralelepípedo



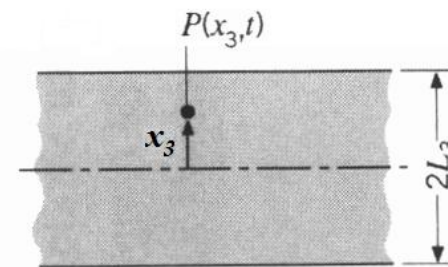
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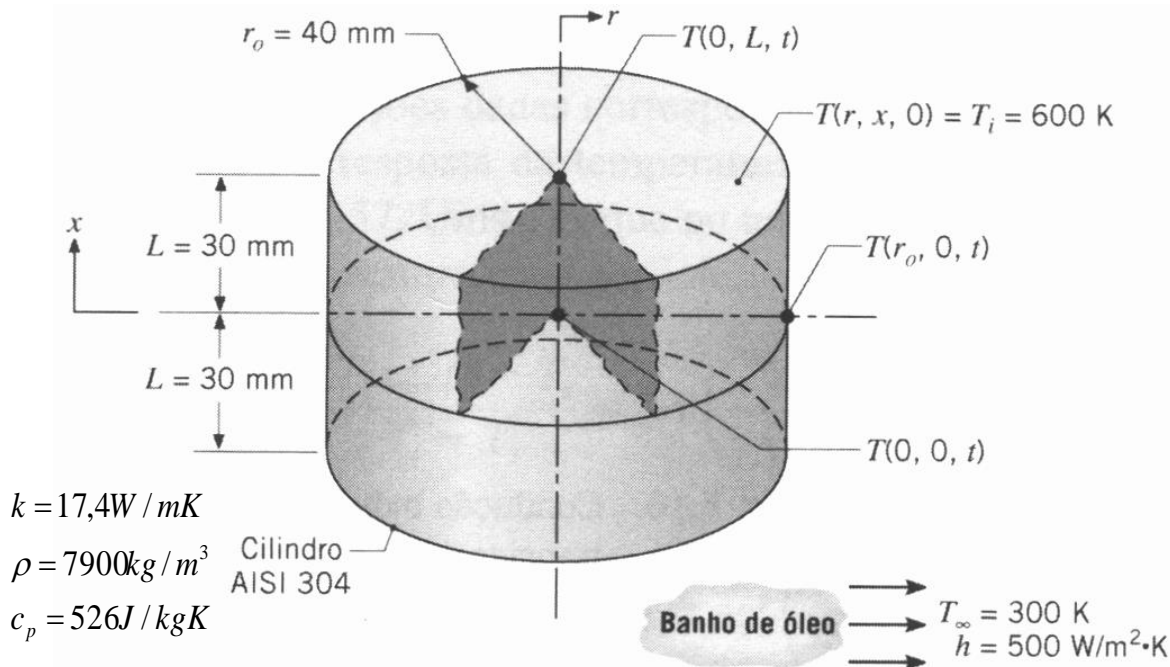


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Efeitos Multidimensionais

Exercício Resolvido 5.7: cilindro de aço a 600K submerso em banho de óleo a 300K



Considerações:

- Condução 2D em r e x
- Propriedades constantes

Pede-se:

- T no centro do cilindro em $t=3\text{min}$
- T no centro da face circular em $t=3\text{min}$
- T na meia-altura da face lateral

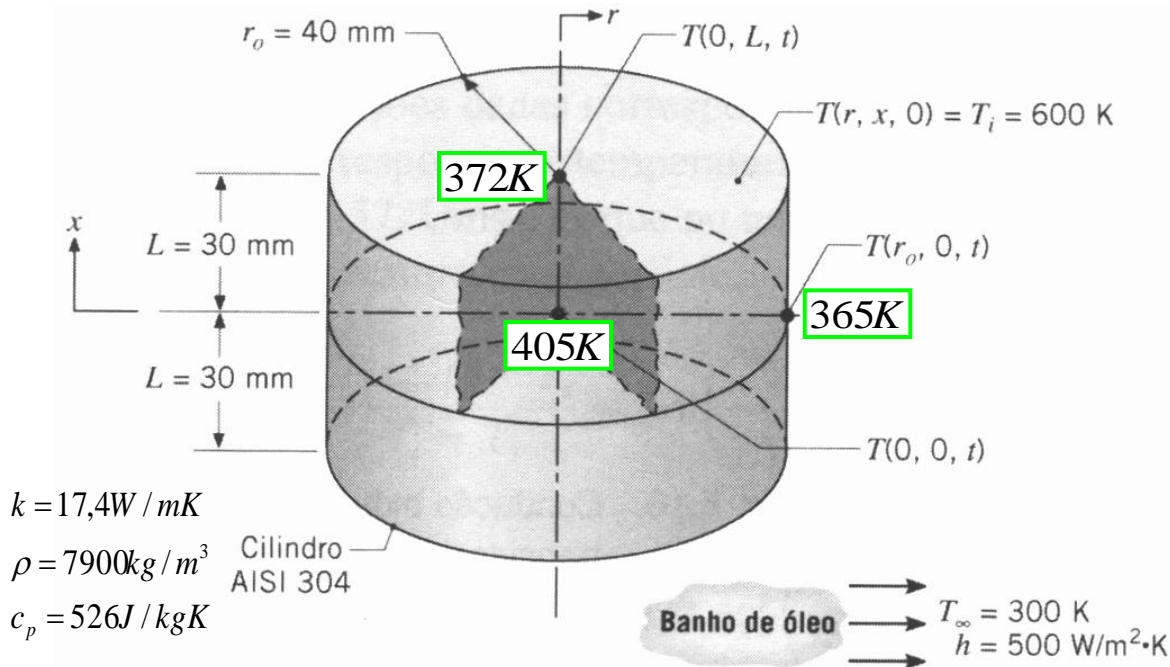
Efeitos Multidimensionais



Exercício Resolvido 5.7: cilindro de aço a 600K submerso em banho de óleo a 300K

Efeitos Multidimensionais

Exercício Resolvido 5.7: cilindro de aço a 600K submerso em banho de óleo a 300K



$$P(0,3 \text{ min}) = 0,6359$$

$$C(0,3 \text{ min}) = 0,5485$$

$$P(L, 3 \text{ min}) = 0,4367$$

$$C(r_o, 3 \text{ min}) = 0,3381$$

Considerações:

- Condução 2D em r e x
- Propriedades constantes

QUAL O PONTO MAIS FRIO DO CILINDRO EM $t=3\text{MIN}$?

Efeitos Multidimensionais

TABLE 5.1 Coefficients used in the one-term approximation to the series solutions for transient one-dimensional conduction

Bi^a	Plane Wall		Infinite Cylinder		Sphere	
	ζ_1 (rad)	C_1	ζ_1 (rad)	C_1	ζ_1 (rad)	C_1
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.6	0.7051	1.0814	1.0184	1.1345	1.2644	1.1713
0.7	0.7506	1.0919	1.0873	1.1539	1.3525	1.1978
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732
2.0	1.0769	1.1785	1.5994	1.3384	2.0288	1.4793
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227

$$C_{1c} = \frac{(1,1494-1)}{(2-1)} (1,3384-1,2071) + 1,2071 = 1,2267$$

$$\xi_{1c} = \frac{(1,1494-1)}{(2-1)} (1,5994-1,2558) + 1,2558 = 1,3071$$